Weak* Closures and Derived Sets for Convex Sets in Dual Banach Spaces

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Weak* sequential closures play an important role in many problems. For example, the conditions for regularizability of unbounded linear operators which are inverses to bounded linear operators can be stated using weak* sequential closures of images of dual operators.

S. Banach and S. Mazurkiewicz started to develop the theory of weak* sequential closures (weak* derived sets) in 1929-1932. I plan to describe the history of the topic and its applications. The main new result: For every nonreflexive Banach space X and every countable successor ordinal α, there exists a convex subset A in X^* such that α is the least ordinal for which the weak* derived set of order α coincides with the weak* closure of A. This result extends the previously known results on weak* derived sets by Ostrovskii (2011) and Silber (2021).

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